Exam. Code : 211003

Subject Code : 4967

M.Sc. Mathematics 3rd Semester (Batch 2020-22)

MATH-572 : TOPOLOGY—I

Time Allowed—3 Hours] [Maximum Marks—100

Note :— Attempt **FIVE** questions in all, selecting at least **ONE** question from each Section. The **fifth** question may be attempted from any Section. All questions carry equal marks.

SECTION—A

1. (i) Let X be a non-empty set and let B* be a non-empty collection of subsets of X. Then show that B* is a subbase for a unique topology T on X. 10

   (ii) Define a second countable space and separable space. Show that a metric space is second countable if and only if it is separable. 10

2. (i) Show that the closure of a set is union of the set and its derived set. 5

   (ii) Show that every closed subspace of a Lindelof space is a Lindelof space. 5

   (iii) Let (X, T) be a topological space and (Y, T_Y) be a subspace of (X, T). State and prove a necessary and sufficient condition for every T_Y-open set to be T-open. 10

SECTION—B

3. (i) Let Y be a topological space. Prove that the union of any family of connected subsets of Y with non-empty intersection is connected. 10

(ii) Define component of a space. Prove that components of a space are closed. Also find component of $q \in \mathbb{Q}$, for set of rationals. 5

(iii) Let $X = A \cup B$ where A, B are both open or both closed in X. Let $f : X \to Y$ be such that $f_{|A}$ and $f_{|B}$ are continuous. Then prove that f is continuous. 5

4. (i) Let X be a space. Prove that the following statements are equivalent :

(a) X is connected space

(b) There is no proper clopen (both closed and open) subset of X

(c) There is no continuous surjection from X to 2. 10

(ii) Prove that $\mathbb{R}^2-\{(0,0)\}$ is connected. Using this or otherwise, prove that the plane and real line cannot be homeomorphic. 10

SECTION—C

5. (i) Define product space. Prove that in a product space $\Pi_\alpha Y_\alpha$ if $A_\alpha \subset Y_\alpha \forall \alpha$ then $\overline{\Pi_\alpha A_\alpha} = \Pi_\alpha \overline{A}_\alpha$. 10

(ii) Prove that the product space $X \times Y$ is connected if and only if topological space X and Y are connected. 10

6. (i) If (X_1, τ_1) and (X_2, τ_2) are any two topological spaces, then prove that the collection $B = \{G_1 \times G_2 : G_1 \in \tau_1, G_2 \in \tau_2\}$ is a base for some topology on $X = X_1 \times X_2$. 10

(ii) Suppose that X is a space with the discrete topology and $\mathcal{R}$ is an equivalence relation on X. Prove that the quotient topology on $X/\mathcal{R}$ is discrete. 10

SECTION—D

7. (i) State and prove Tietze extension theorem. 15

(ii) Let (X, τ_1) be a topological space and (X, τ_2) be a Hausdorff space and $f : X \to Y$ be a one-one continuous map then prove that (X, τ_1) is also a Hausdorff space. 5

8. (i) Prove that a topological space (X, τ) is a T_o space iff the closure of distinct points are distinct. 10

(ii) Prove that a topological space (X, τ) is normal iff for any τ-closed set F and open set G containing F there exists an open set V such that $F \subset V$ and $\overline{V} \subset G$. 10